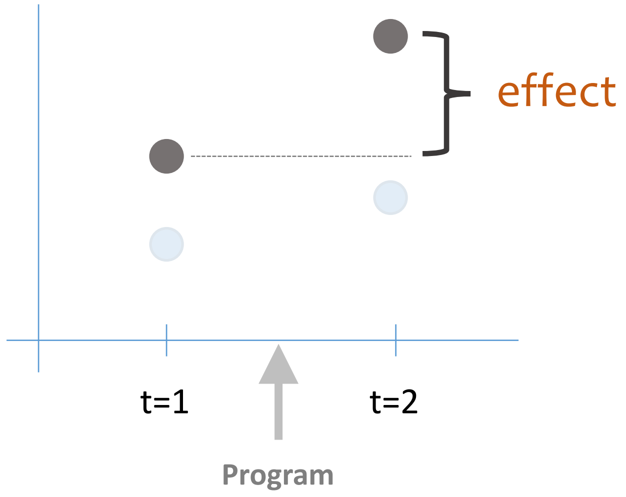
COUNTERFACTUAL ESTIMATORS

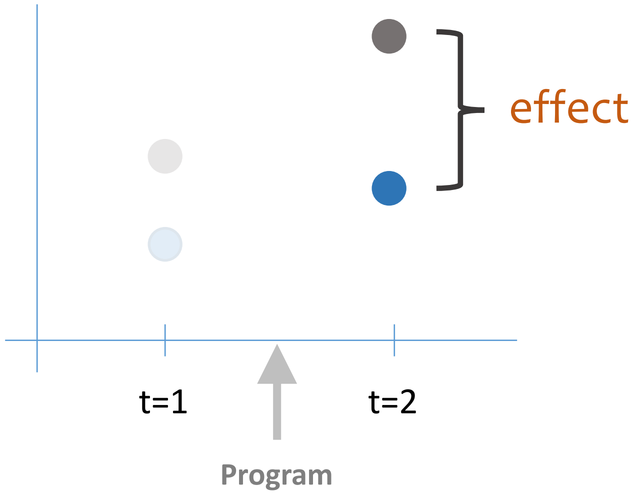
3 valid counterfactuals:

Each variety of counterfactual has a different formula for the program effect estimate.

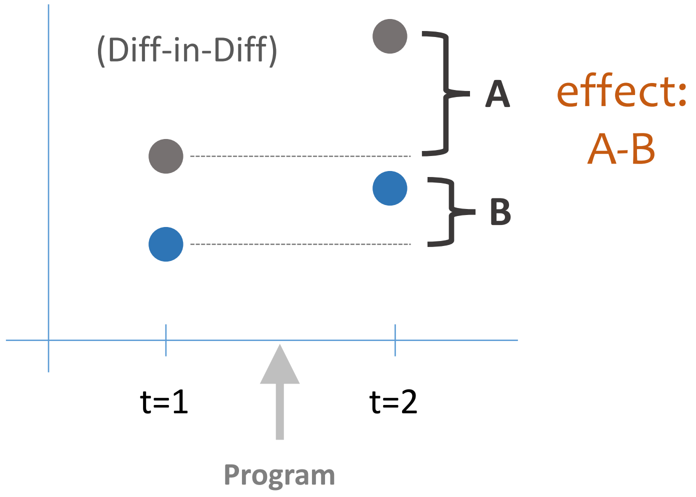
PRE-POST REFLEXIVE
Estimator
effect = T2-T1



POST-ONLY
Estimator
effect = T2-C2



PRE-POST W COMPARISON
Estimator
effect = (T2-T1) – (C2-C1)



● Treatment Groups, **T1**=before, **T2**=post-program measure

● Control Groups, **C1**=before, **C2**=post-program measure

Recall from Unit on Dummy Variable Models

(basic set-up for a comparison of group means in regression)

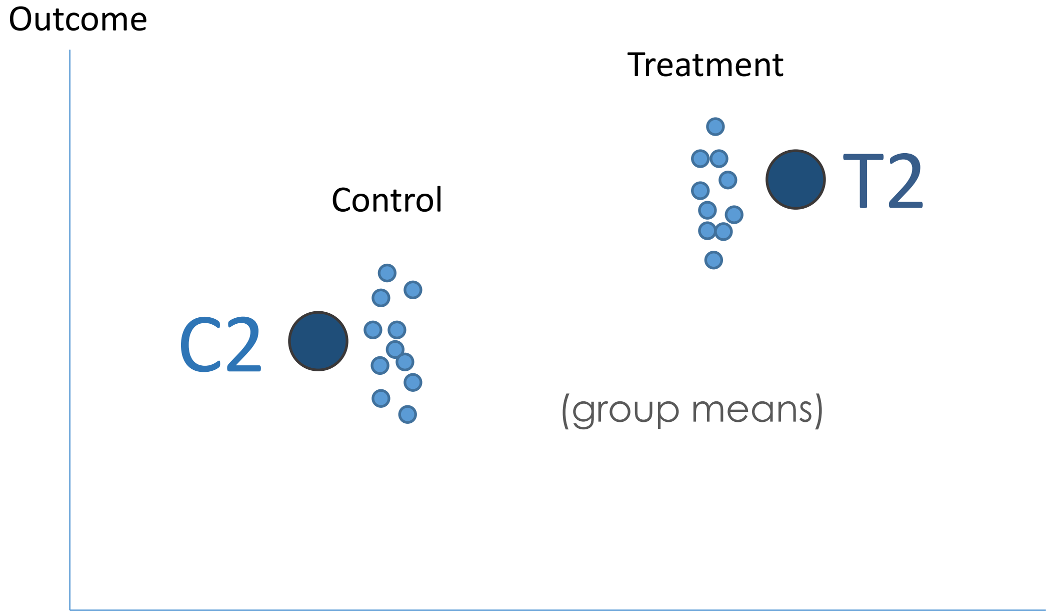
b_0 will measure Y-bar of the omitted group C2

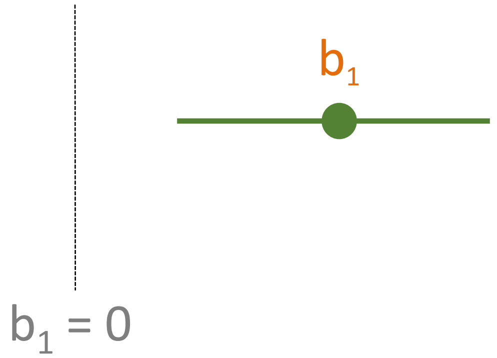
b_0 +b_1 represents the Y-bar of T2

$$Y = b_0 + b_1(\text{TreatDummy}) + e$$

$$b_1 = \text{MEAN}_{\text{treat}} - \text{MEAN}_{\text{control}}$$

$$b_1 = \text{T2} - \text{C2}$$



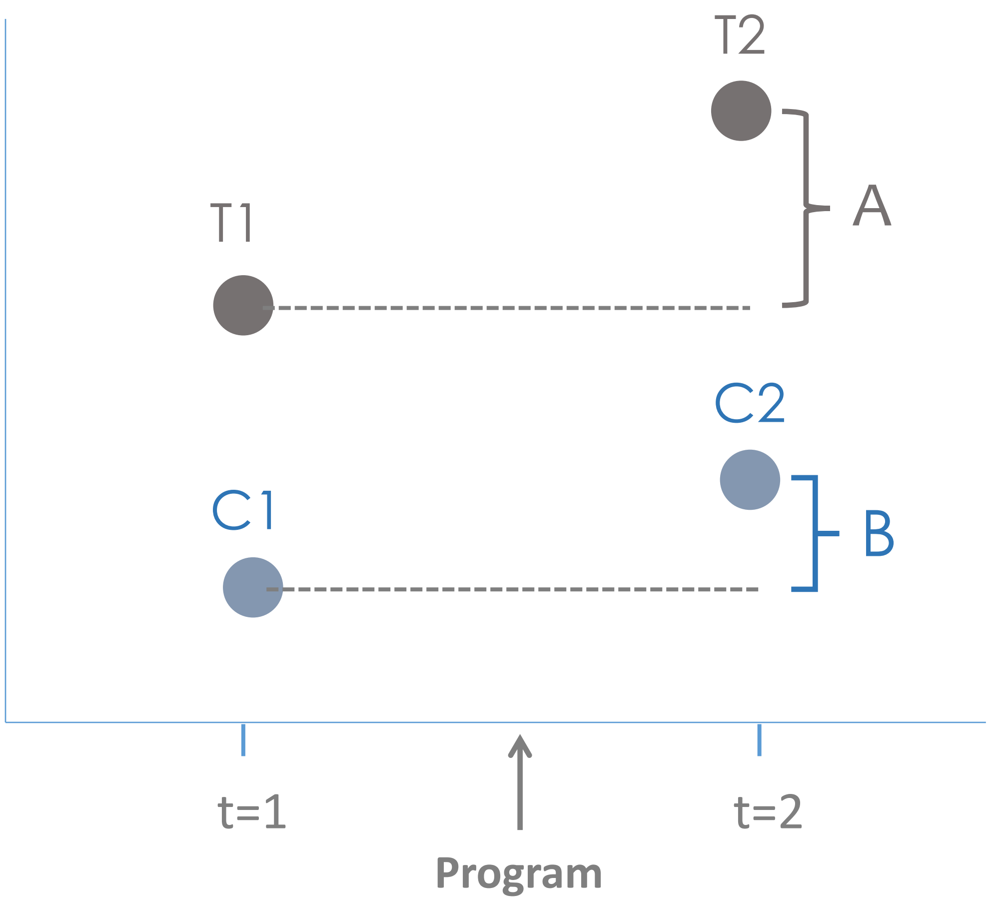


The default hypothesis test in the regression then uses **b_1** to **test for a meaningful difference between T2 and C2**

Difference-in-Difference Estimator

Pre-Post w Comparison
Estimator

effect = **(T2-T1) – (C2-C1)**

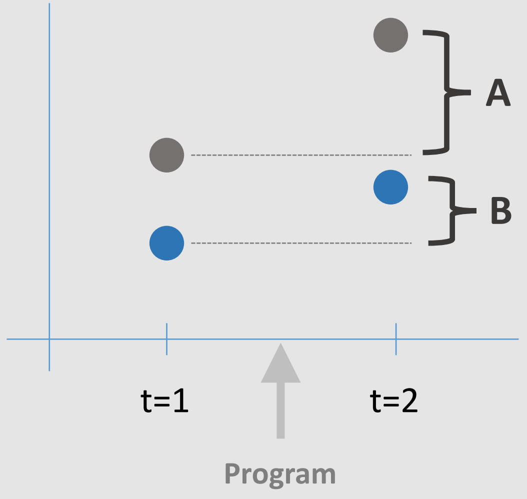


effect:
A-B
(Diff-in-Diff)

Accounts for initial difference in groups and secular trends

Difference-in-Difference Estimator

effect:
A-B

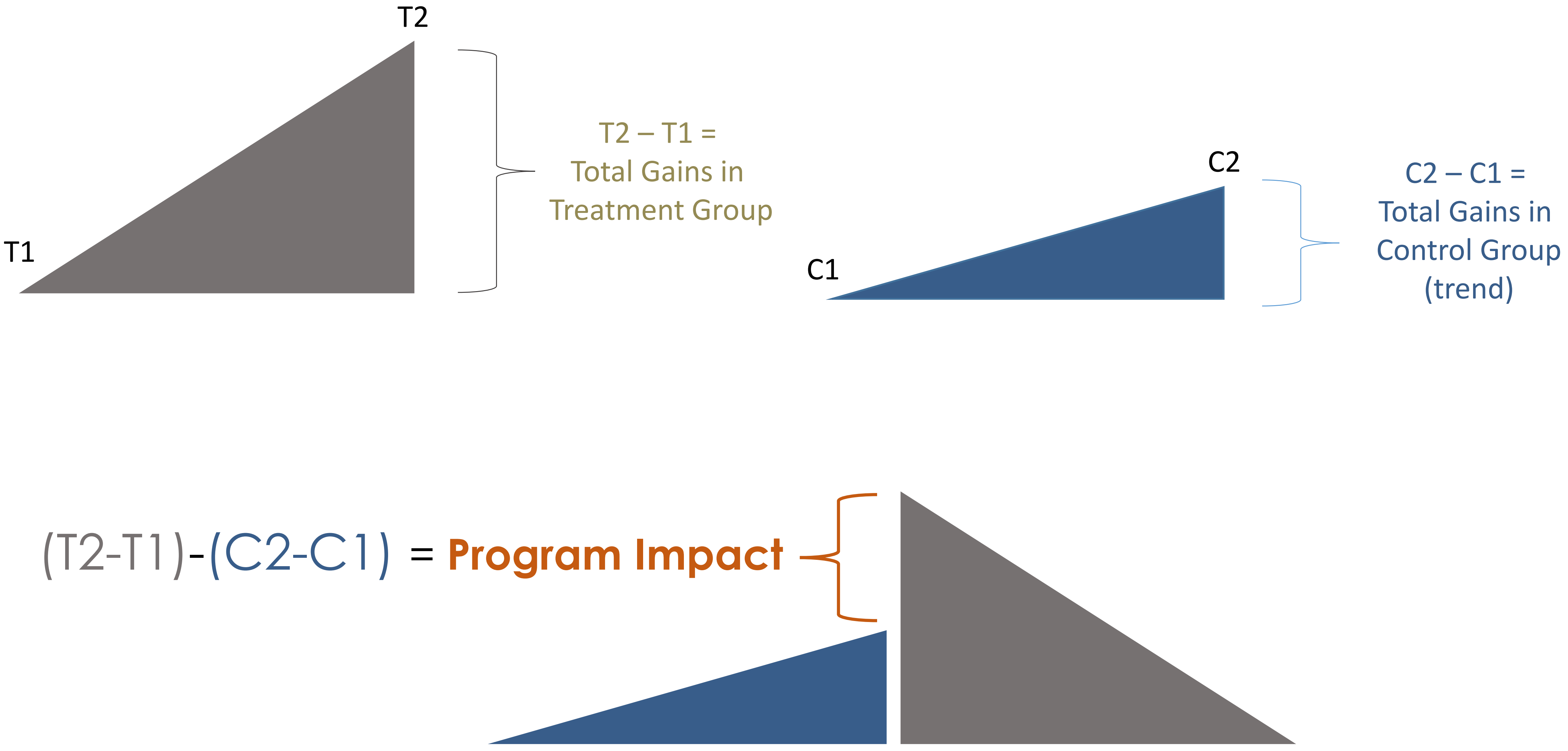


Pre-Post w Comparison

effect = (T2-T1) – (C2-C1)

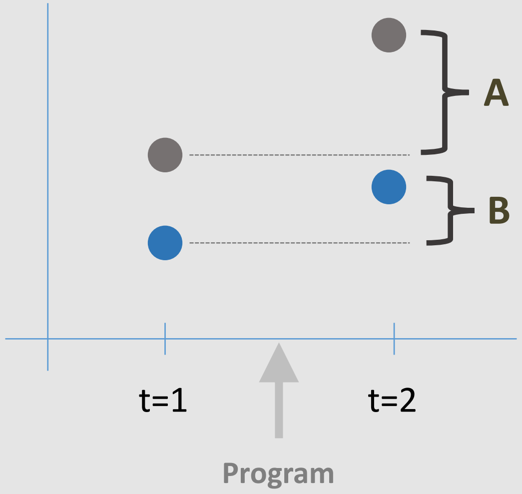
Gains during the program period by treatment group

Gains independent of the program (secular trends)


T2
T1
T2 – T1 =
Total Gains in
Treatment Group
C2
C1
C2 – C1 =
Total Gains in
Control Group
(trend)
(T2-T1)-(C2-C1) = Program Impact

DIFFERENCE-IN-DIFFERENCE ESTIMATOR

effect:
A-B



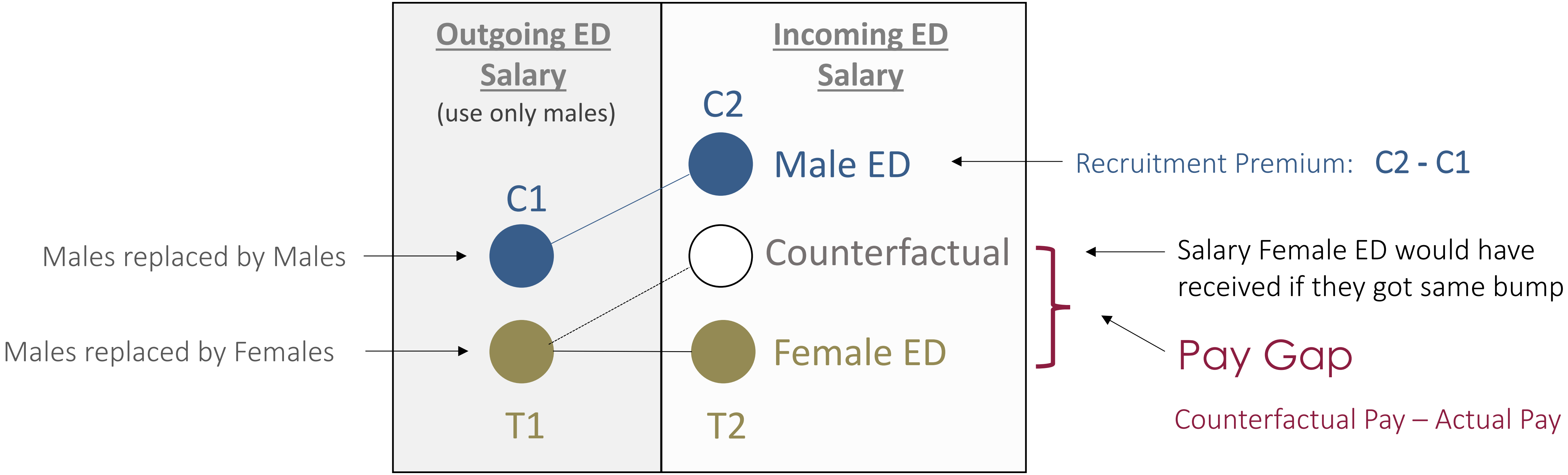
$$\text{effect} = (T2\text{-}T1) - (C2\text{-}C1)$$

<u>Two important things to note:</u>

(1) The groups prior to the treatment are not identical. The difference in difference is robust because it does not require equivalent treatment and "control" (comparison) groups.

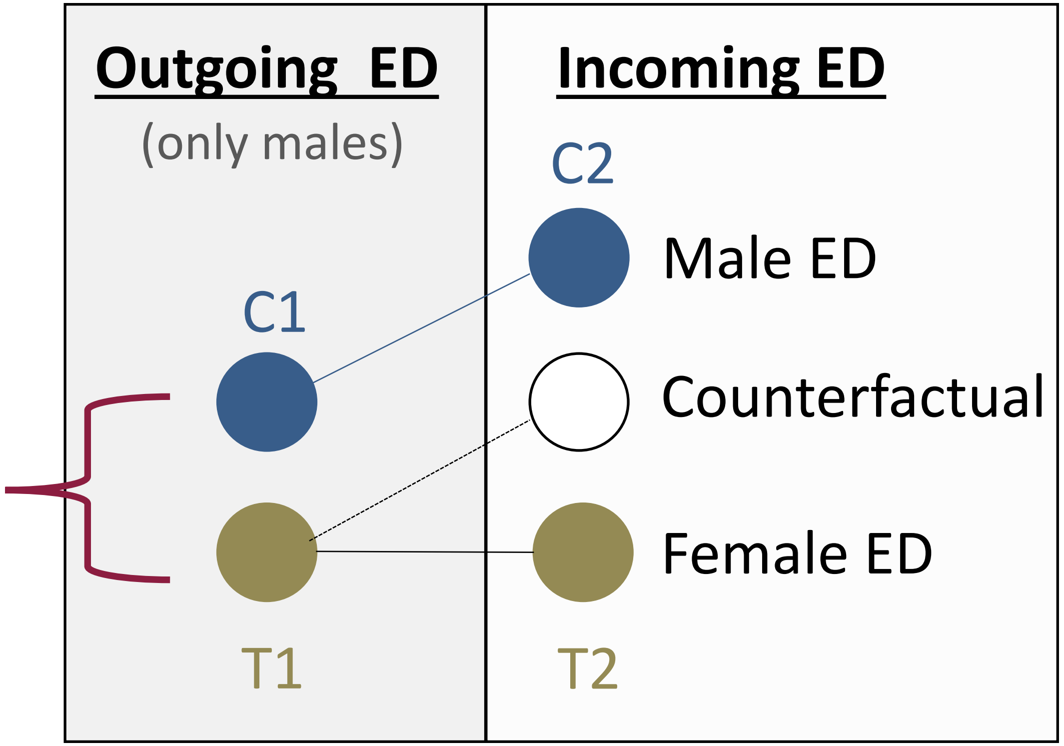
(2) The comparison group measures the changes we would expect to observe in the treatment group over time independent of the treatment. This assumption must be true for the diff-in-diff estimator to be unbiased.

Example: Estimating the gender pay gap for Executive Directors



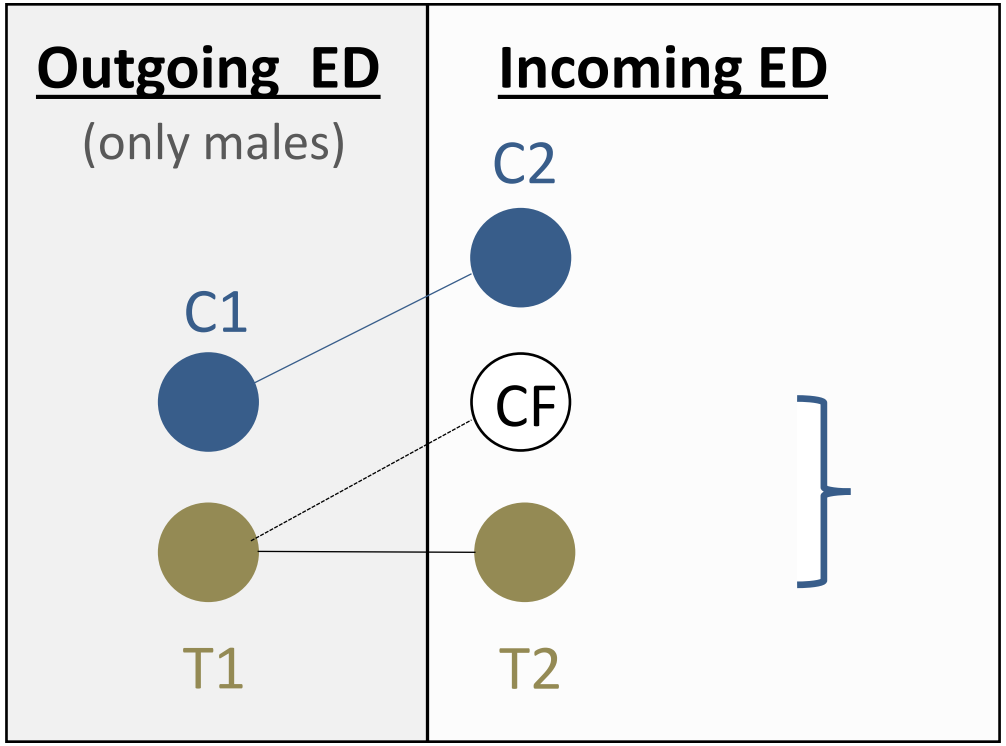
Counterfactual Pay: Baseline Pay (T1) + Typical (male) Recruitment Premium (C2 – C1)

Note that in the diff-in-diff the groups do not have to be identical prior to the treatment (new hire)

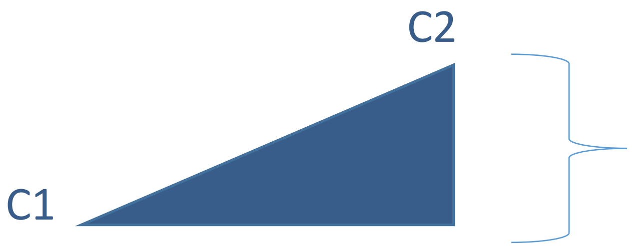


This is the major strength of the estimator.

Also note that we never compare C2 to T2 (the two post-treatment groups)

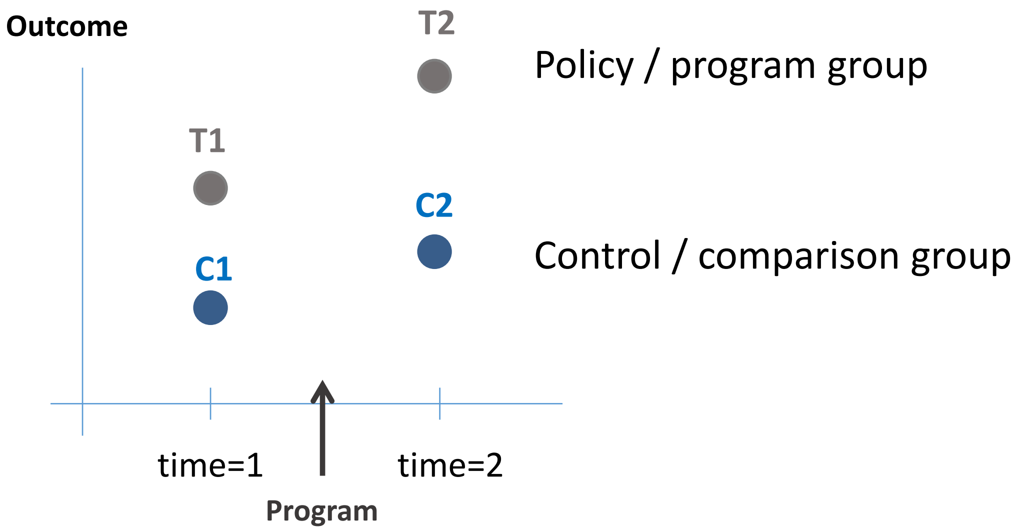


The proper counterfactual is how much would female EDs be getting paid if they were given recruitment premiums as incoming male EDs.



The comparison group is there to capture the trend, which is then used to construct the counterfactual.

“Control” Versus “Comparison” Groups



<u>Single Difference:</u>

Effect = T2 – T1

Effect = T2 – C2

<u>Difference in Difference:</u>

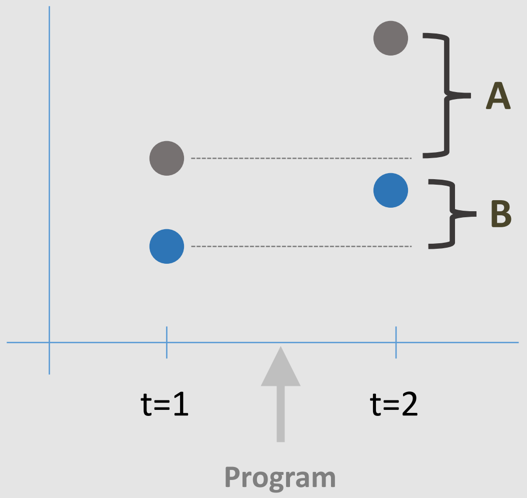
Effect = (T2 – T1) – (C2 – C1)

trend

The term “control group” typically refers to a group created through randomization, so they are assumed to be identical. We sometimes use the terminology “comparison group” to differentiate the these cases. The control groups represents the counterfactual, the comparison group does not (it just captures trend in this case).

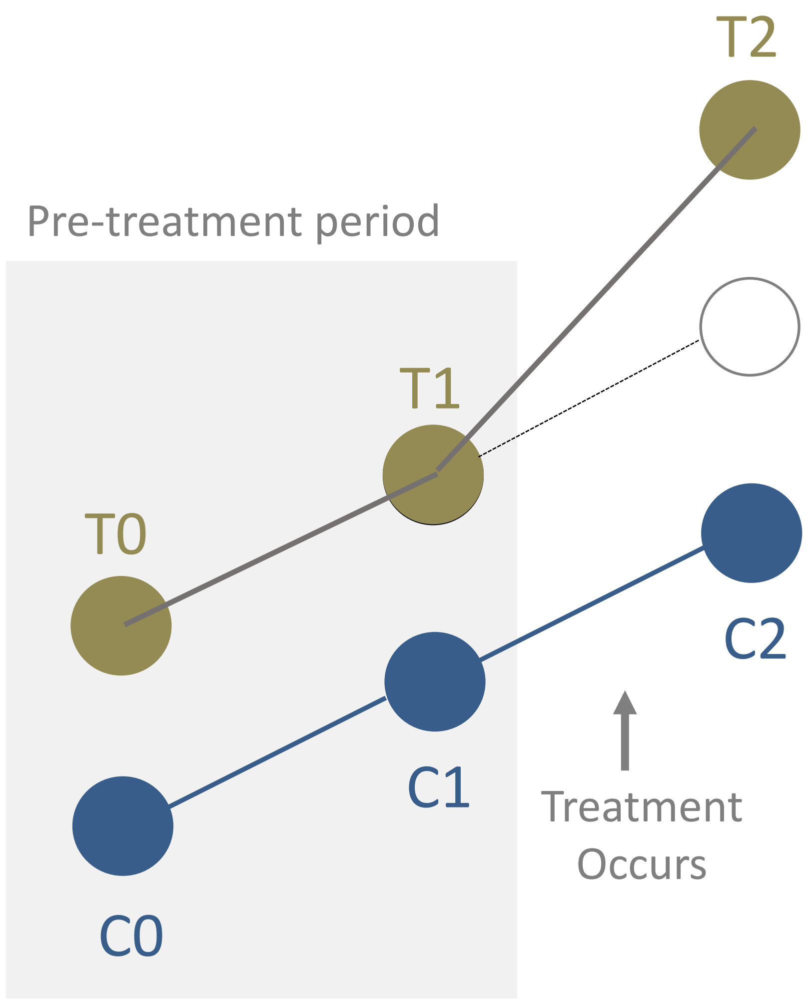
Difference-in-Difference Estimator

effect:
A-B



Parallel lines test:

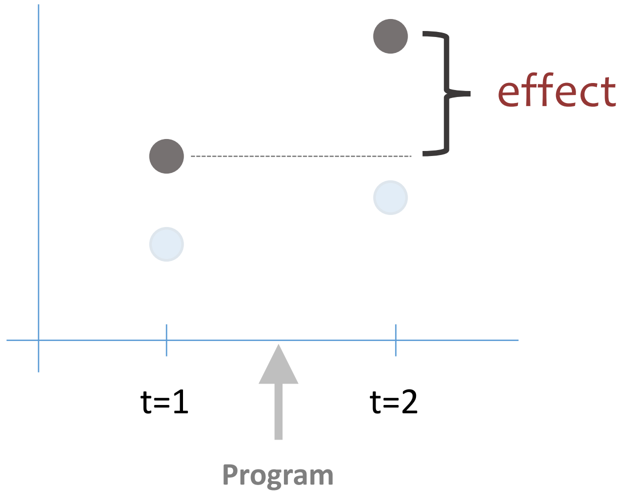
T1-T0 = C1- C0



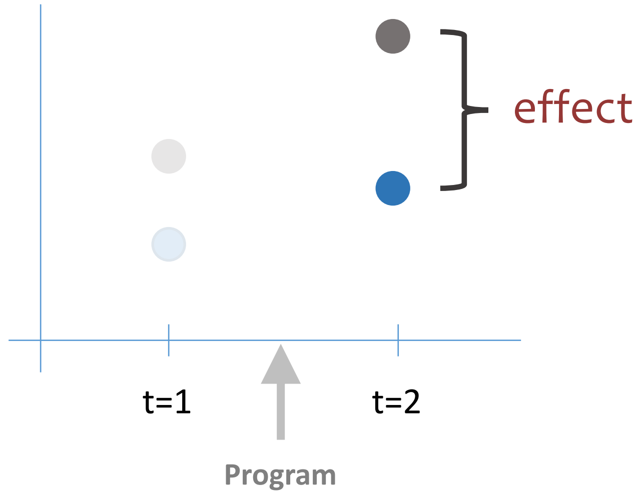
Parallel lines assumption: for the diff-in-diff counterfactual to be valid C2-C1 has to accurately capture the secular trend (gains independent of the treatment). The test of this is comparing rates of change in treatment and comparisons groups prior to the intervention. They should b the same.

2 remaining counterfactuals:

Pre-Post Reflexive
Estimator
effect = T2-T1



Post-Only
Estimator
effect = T2-C2



Pre-Post w Comparison
Estimator
effect = (T2-T1) – (C2-C1)

Validity of the reflexive estimator:

Reflexive (Pre-Post)

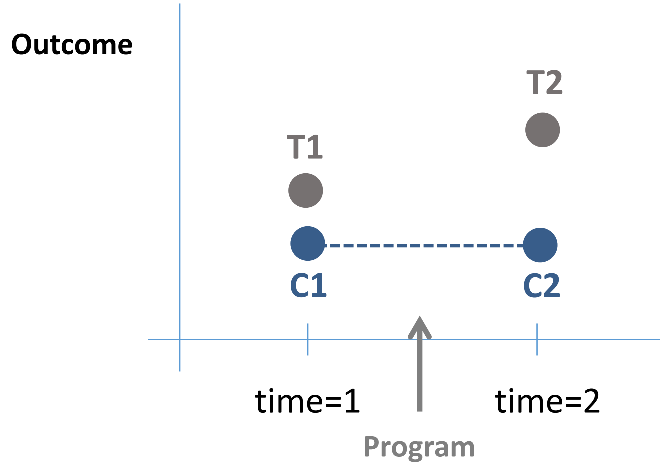
(T2 – T1) – (C2 – C1) = T2 – T1

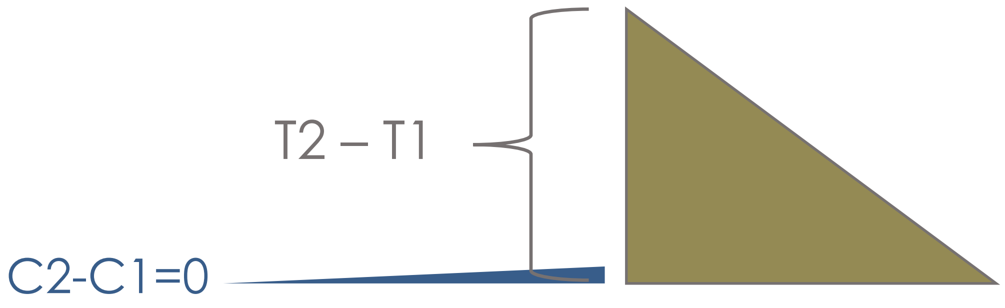
IFF

C2 – C1 = 0

(no trend)

In some special cases we expect there to be no change independent of the treatment. When this is true the reflexive estimator T2-T1 is appropriate and unbiased.





b_0 will measure Y-bar of the omitted group **T1**

$b_0 + b_1$ represents the Y-bar of **T2**

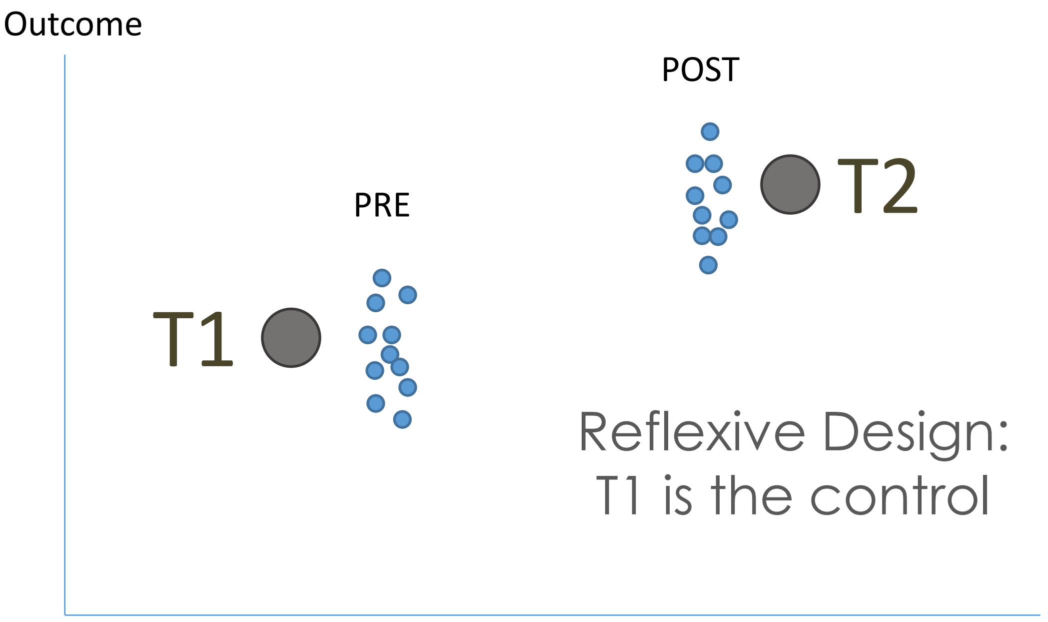
$$Y = b_0 + b_1(\text{TreatDummy}) + e$$

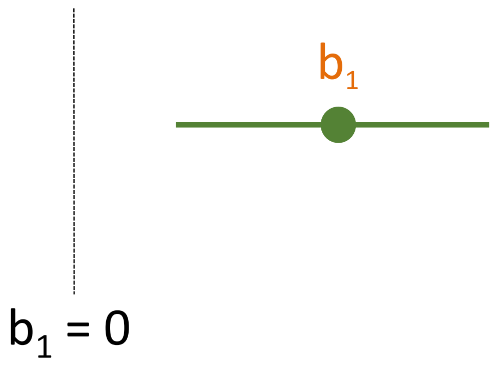
$$b_1 = \text{MEAN}_{\text{treat}} - \text{MEAN}_{\text{control}}$$

$$b_1 = \text{T2} - \text{T1}$$

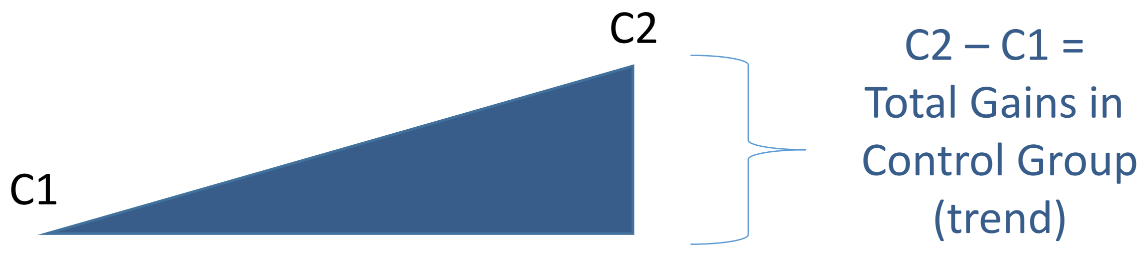
In reflexive models T1 is the counterfactual or control group, otherwise the regression set-up is the same

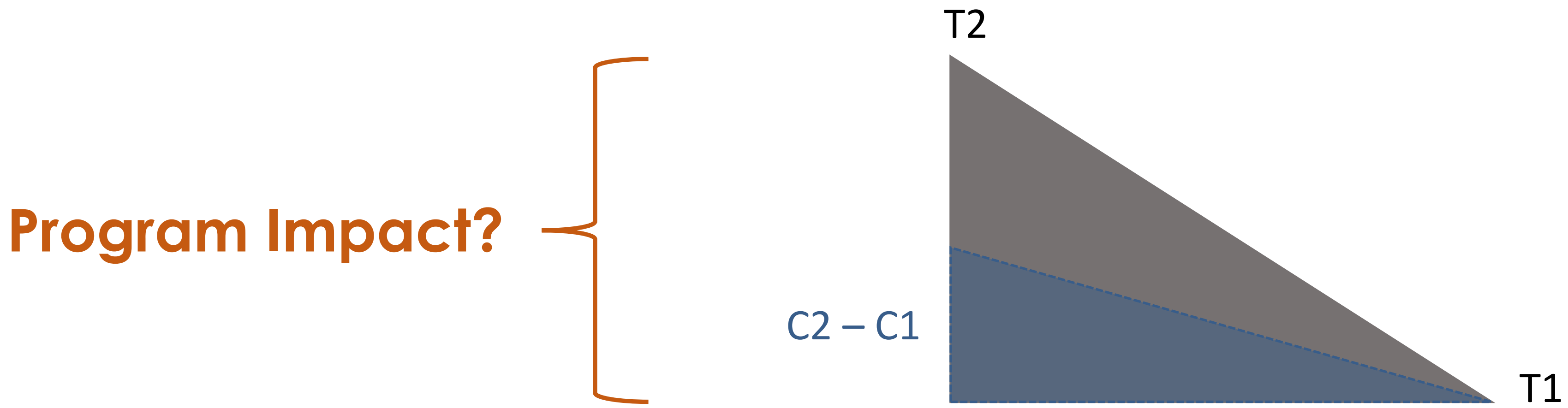
The default hypothesis test in the regression then uses $\mathbf{b_1}$ to test for a meaningful difference between T2 and T1





The comparison group is necessary to capture any changes that we expect during the study period independent of the treatment group. If we fail to account for those changes we will incorrectly attribute all change to the program and over-state program impact.





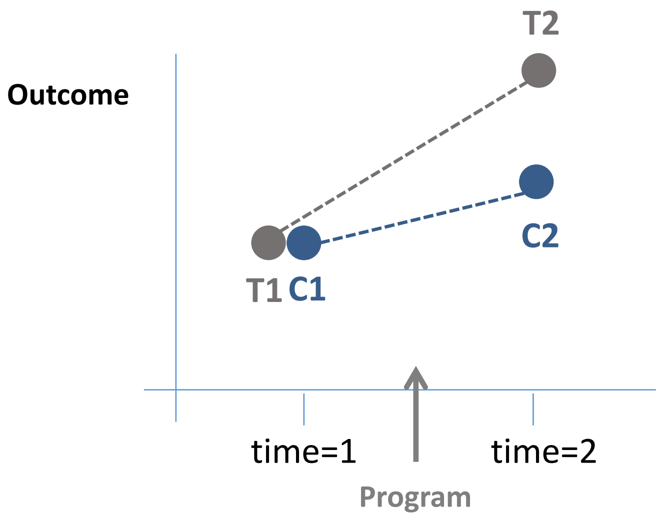
Validity of the post-test only estimator:

Post-Only

(T2 – T1) – (C2 – C1) = T2 – C2

IFF

C1 – T1 = 0
(equivalent at time 1)



If we have confidence that the two groups are identical prior to the treatment, then mathematically T2-C2 will still account for gains independent of the treatment. This condition is necessary for the post-test only estimator to be unbiased.

In experimental design, this is usually accomplished through randomization or lottery.

Observational students typically use matching models to create equivalent groups.

Recall from Unit on Dummy Variable Models

(basic set-up for a comparison of group means in regression)

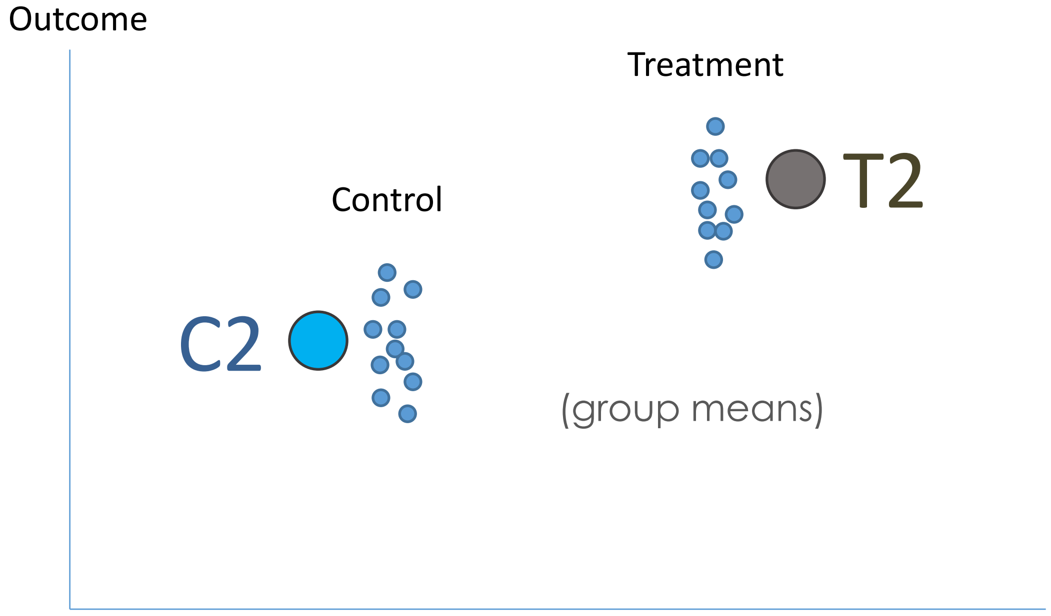
b_0 will measure Y-bar of the omitted group C2

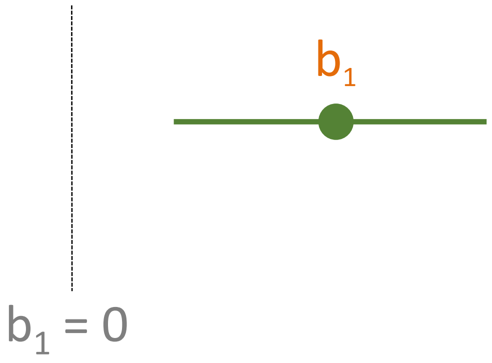
b_0 +b_1 represents the Y-bar of T2

$$Y = b_0 + b_1(\text{TreatDummy}) + e$$

$$b_1 = \text{MEAN}_{treat} - \text{MEAN}_{control}$$

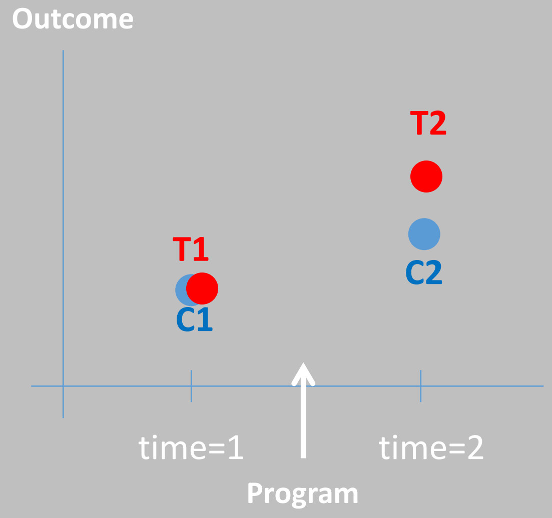
$$b_1 = T2 - C2$$

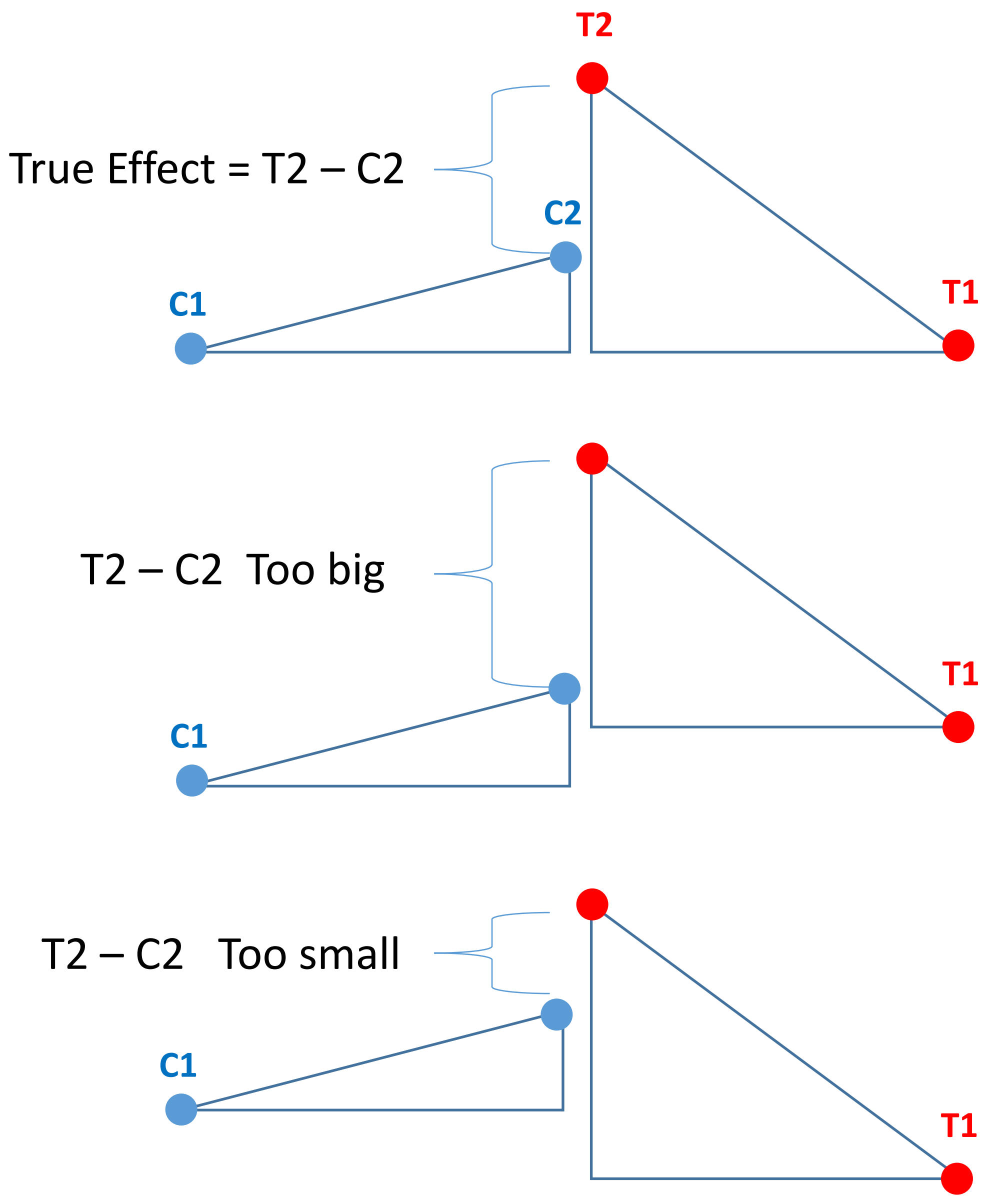




The default hypothesis test in the regression then uses **b_1** to **test for a meaningful difference between T2 and C2**

Post-Test Only Measure





If C1 = T1

Measured effect accurately represents program impact

C1 < T1 biased

Measured effect overstates program impact

C1>T1 biased

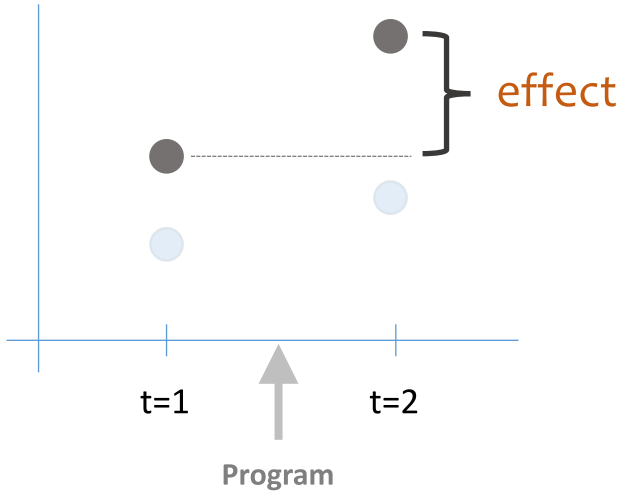
Measured effect understates program impact

HYPOTHESIS TESTING IN THE COUNTERFACTUAL FRAMEWORK

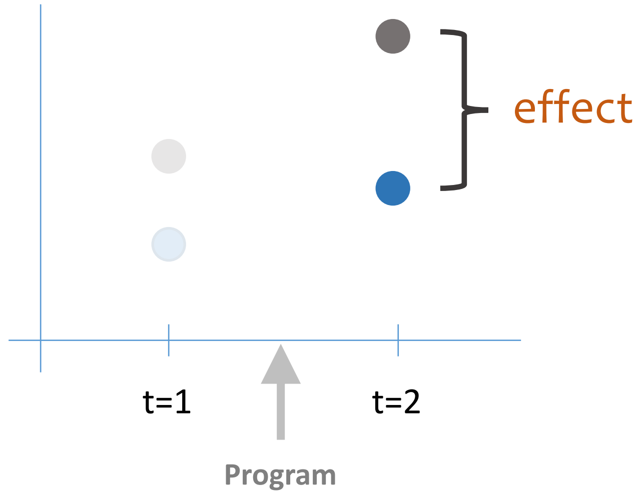
3 valid counterfactuals:

Each variety of counterfactual has a different formula for the program effect estimate.

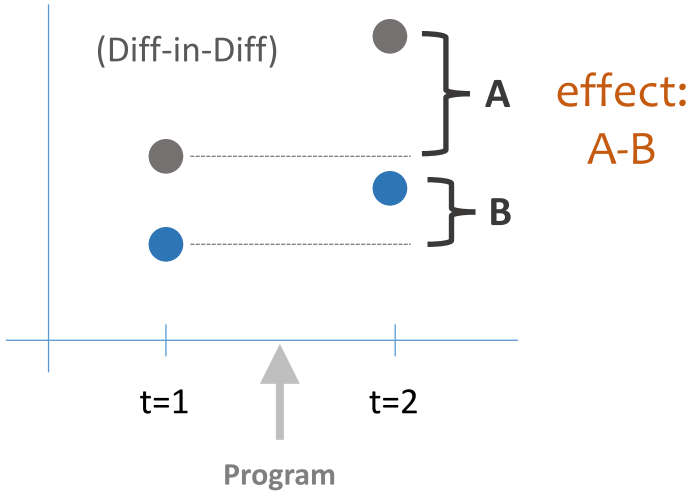
PRE-POST REFLEXIVE
Estimator
effect = T2-T1



POST-ONLY
Estimator
effect = T2-C2



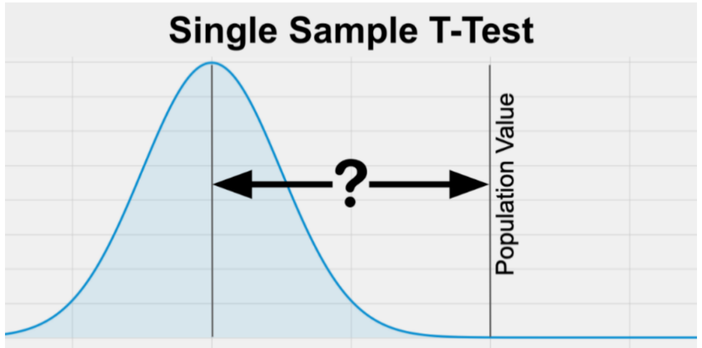
PRE-POST W COMPARISON
Estimator
effect = (T2-T1) – (C2-C1)



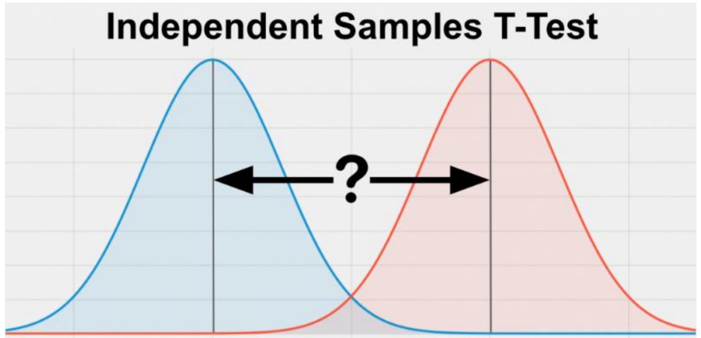
Treatment Groups, **T1**=before, **T2**=post-program measure

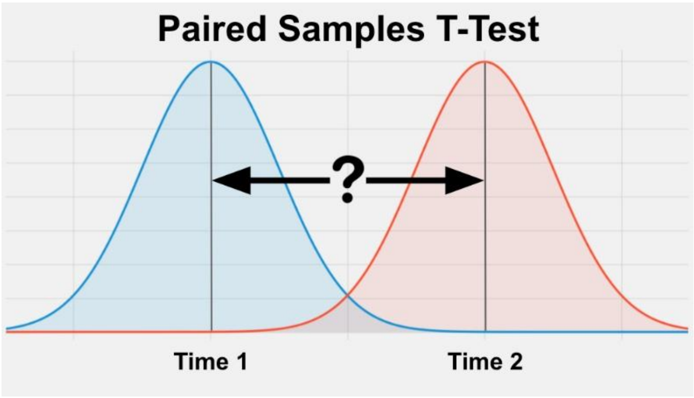
Control Groups, **C1**=before, **C2**=post-program measure

T-Tests are always tests of whether the difference is equal to zero



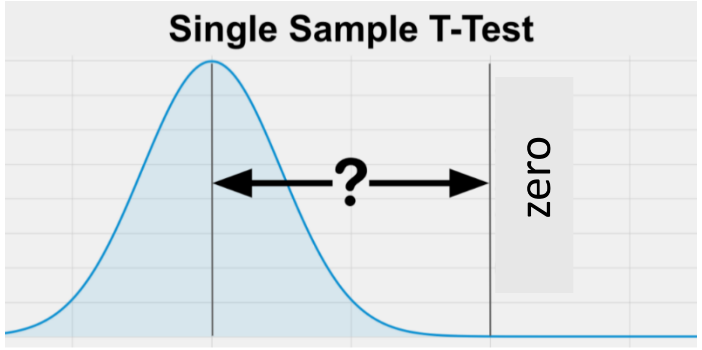
Diff: X - U





Diff: X2-X1

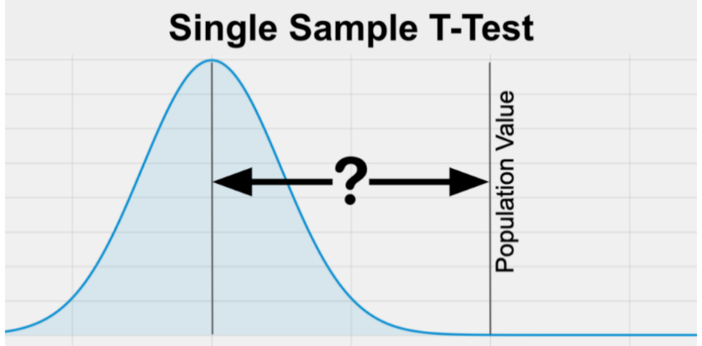
Diff: T2-T1

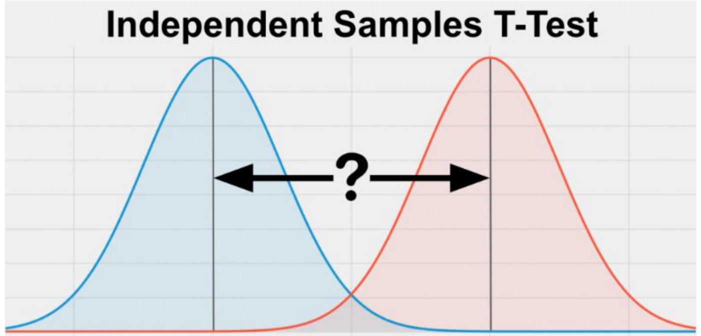


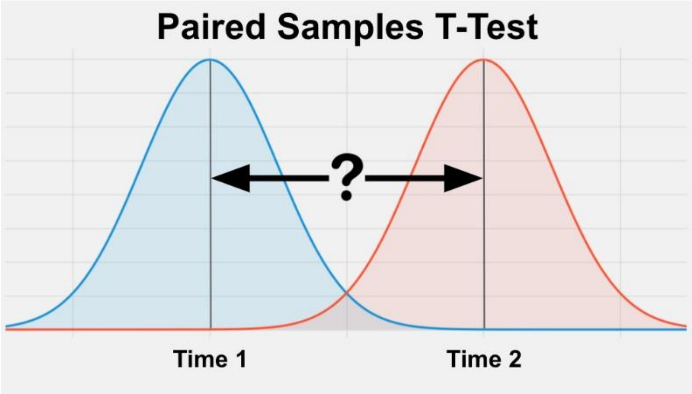
Diff

Δ=0

Estimators as T-Tests







Diff: T2-T1

PRE-POST W COMPARISON

Estimator

effect = (T2-T1) – (C2-C1)

POST-ONLY

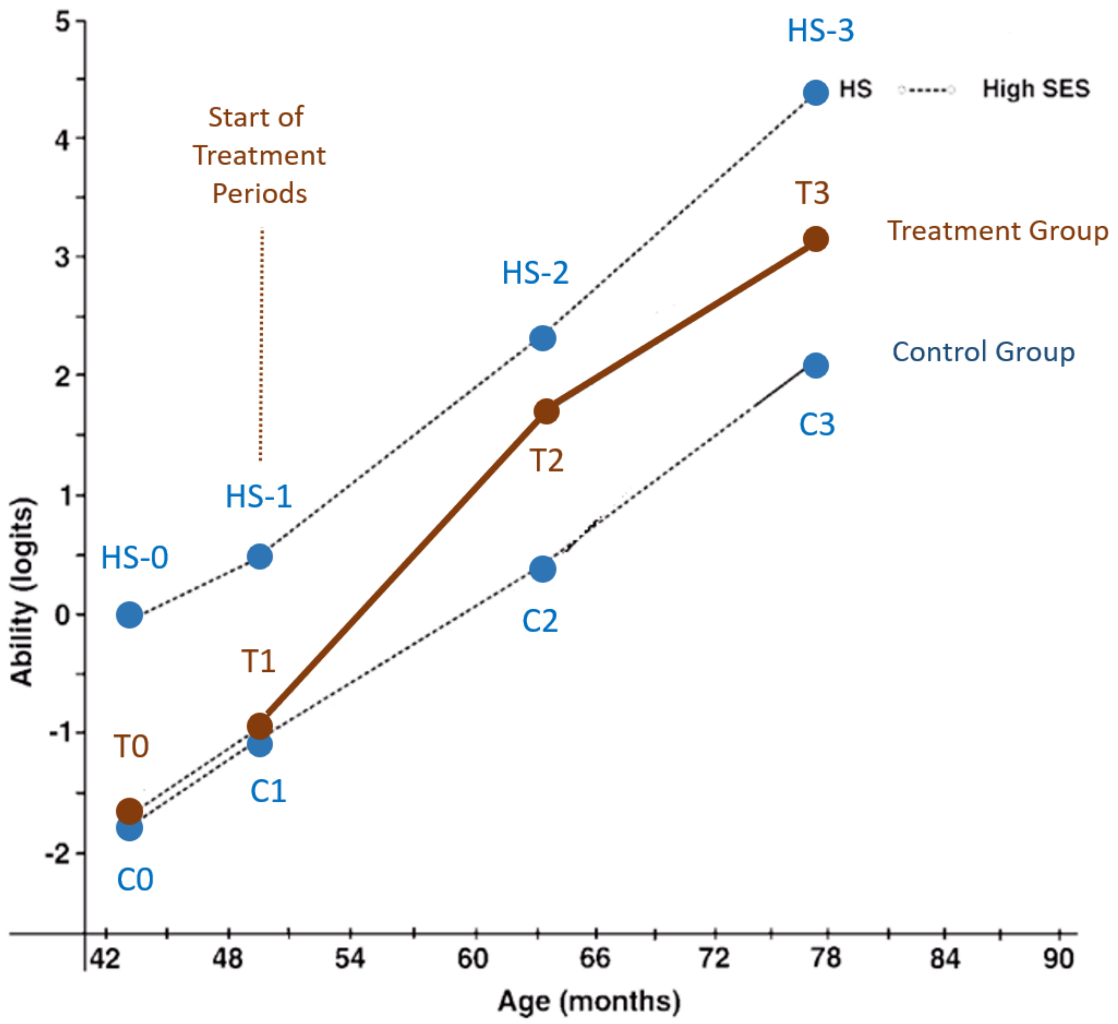
Estimator

effect = T2-C2

PRE-POST REFLEXIVE

Estimator

effect = T2-T1



HS-3
HS
High SES
Start of
Treatment
Periods
T3
Treatment Group
HS-2
Control Group
C3
T2
HS-1
HS-0
C2
T1
T0
C1
C0
5
4
3
2
1
0
-1
-2
Ability (logits)
42
48
54
60
66
72
78
84
90
Age (months)

One-sample t-test (scalar null)

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

Where:

- $\bar{x}$ = sample mean
- μ_0 = null (hypothesized) population mean (a scalar, e.g., 0, 10, or 100)
- s = sample standard deviation
- n = sample size

Paired t Tests

H_0: $\mu_{before} = \mu_{after}$
H_a: $\mu_{before} \neq \mu_{after}$

$$t = \frac{\bar{d}}{s/\sqrt{n}}$$

Patient	Before	After	difference
1	120	122	-2
2	122	120	2
3	143	141	2
4	100	109	-9
5	109	109	0

Example: Before and after medicine BP was measured. Is there a **difference** at 95% confidence level?

$\bar{d}$ = -1.4 , s = 4.56 , n = 5

$t_{cal.}$ = 1.4/2.04 = - 0.69

SD of the difference

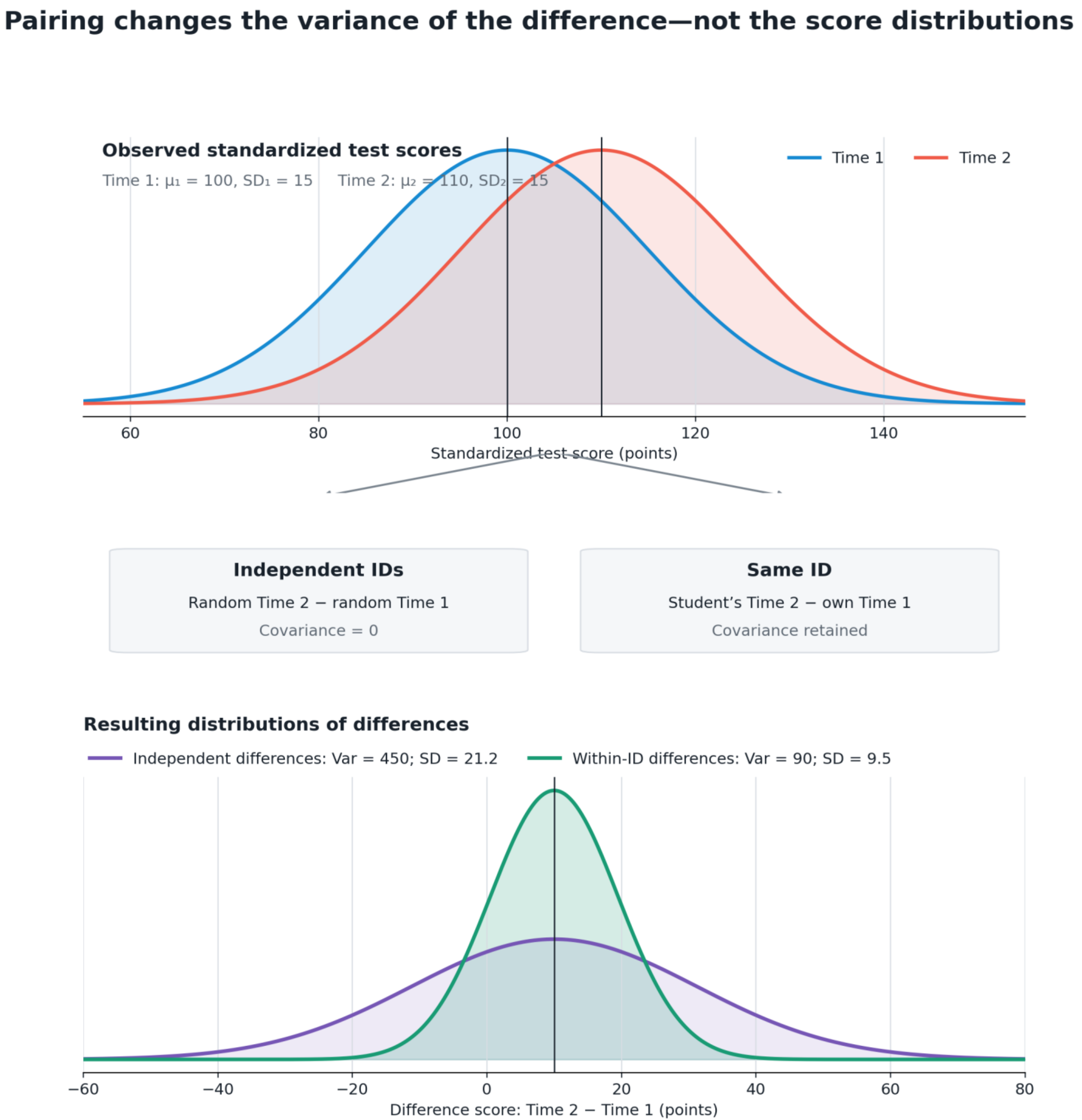
Main Formula (Equal Variances Assumed)

- **t-statistic:**
$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$
- **Pooled Variance (s_p^2):** (Weighted average of sample variances)
$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

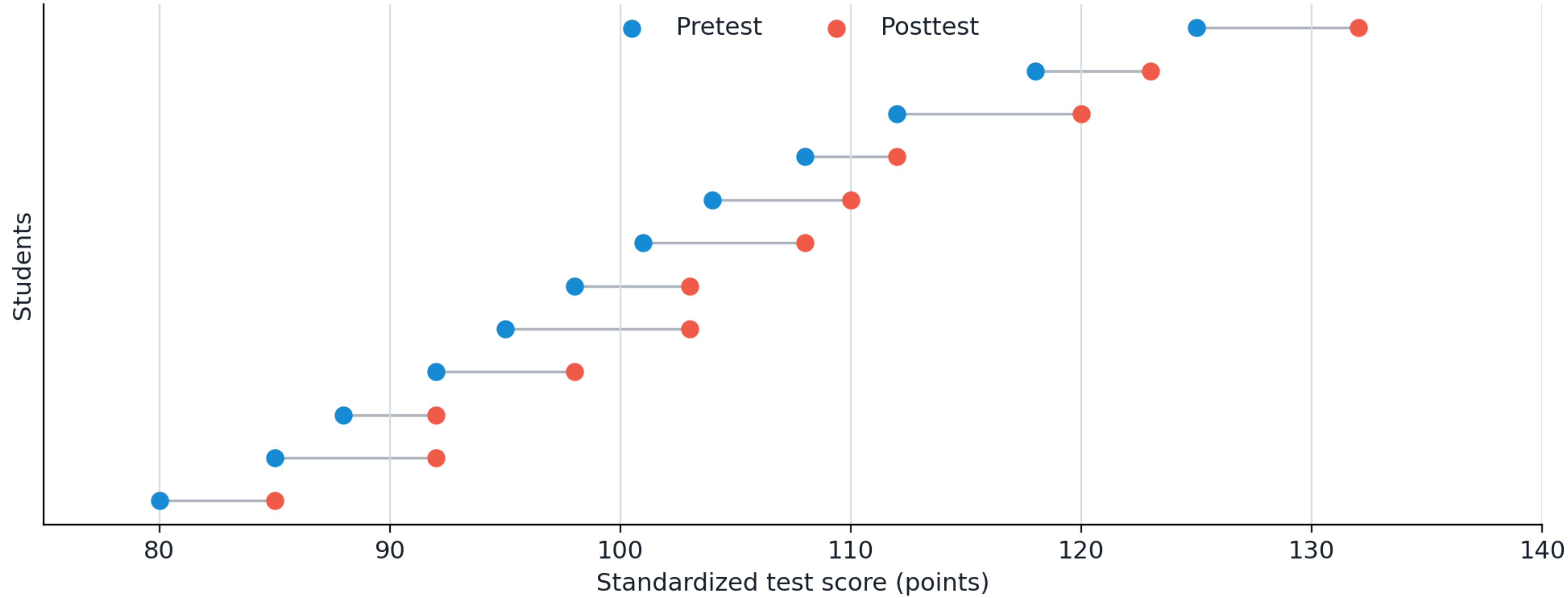
What the Symbols Mean

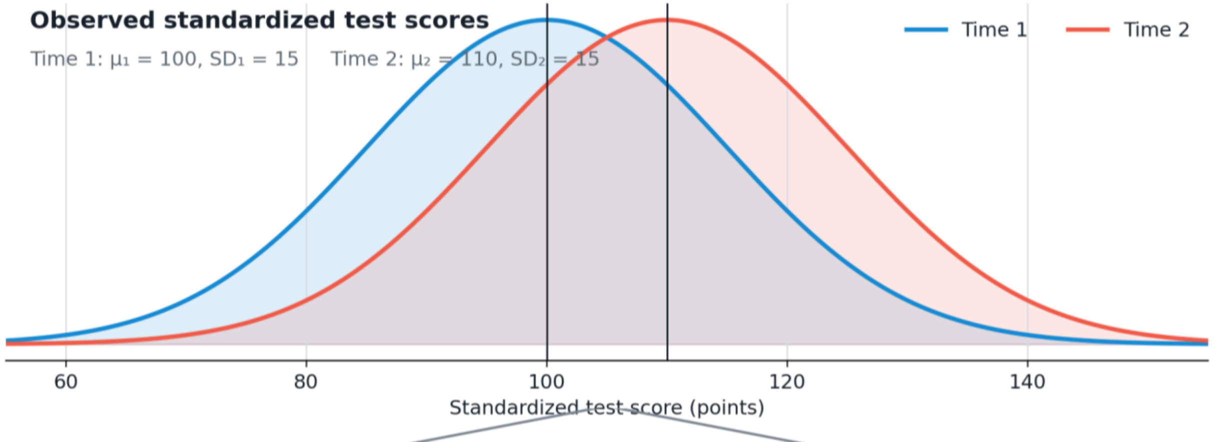
- $\bar{x}_1, \bar{x}_2$: Sample means of the two groups.
- s_1^2, s_2^2: Sample variances of the two groups.
- n_1, n_2: Sample sizes of the two groups.
- s_p: The pooled standard deviation (square root of s_p^2).

Panel data / the reflexive estimator / paired t-tests (multiple observations per ID) have huge statistical power advantages

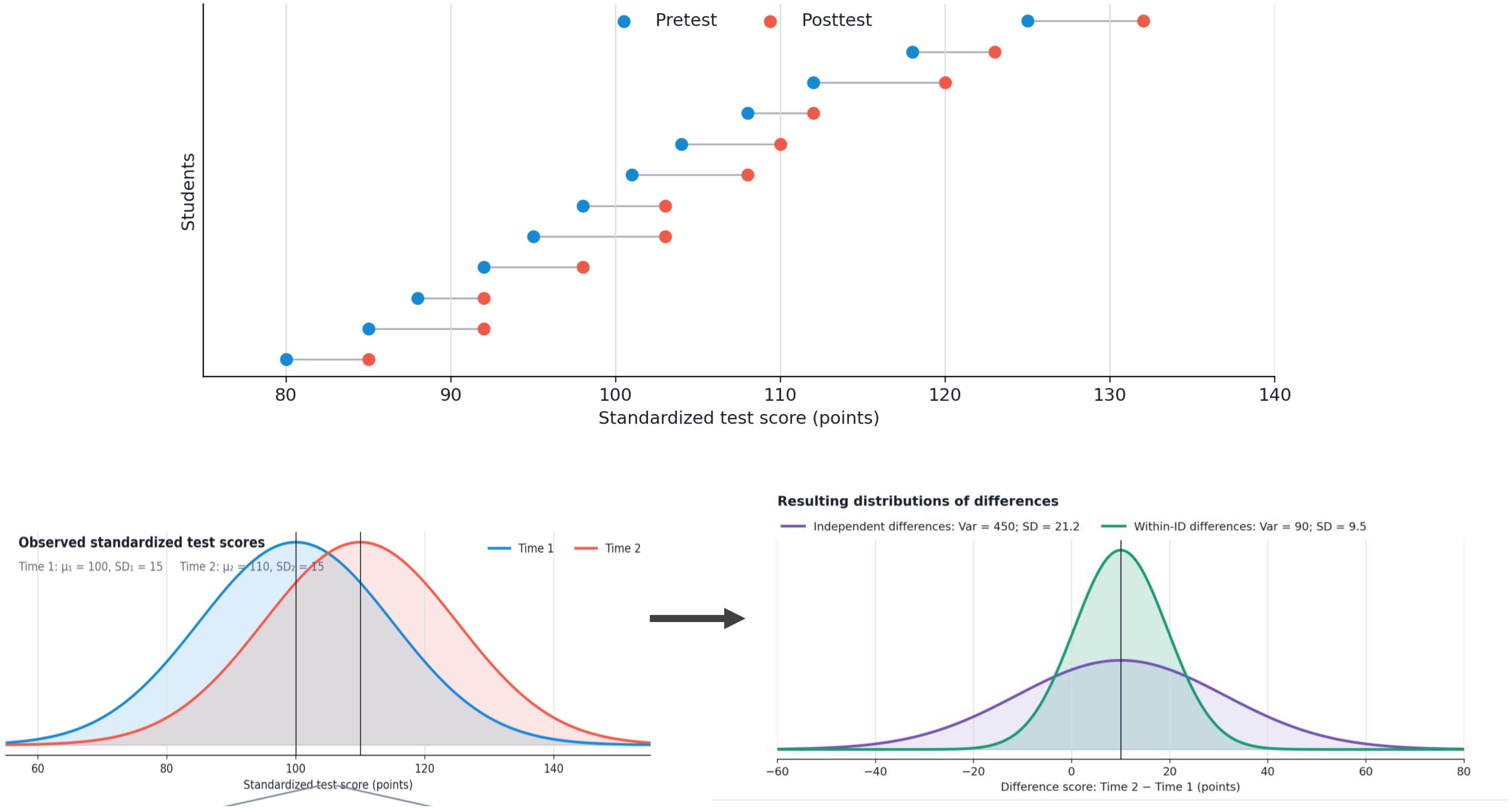


Paired test scores: wide scores, tightly clustered gains

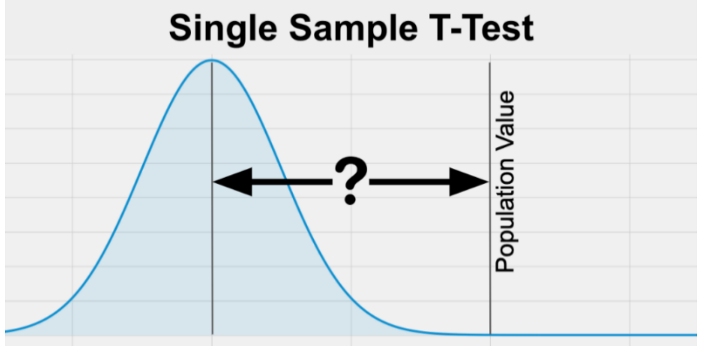




Paired test scores: wide scores, tightly clustered gains

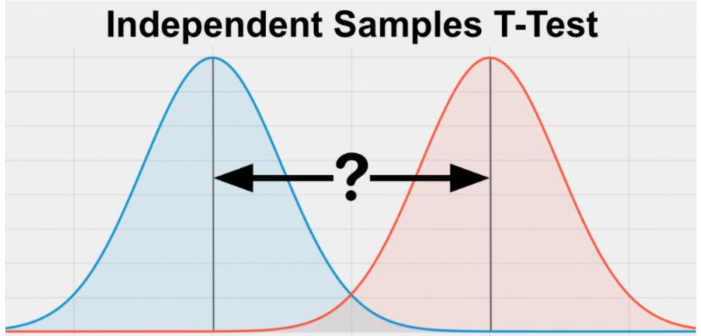


Formulas



$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

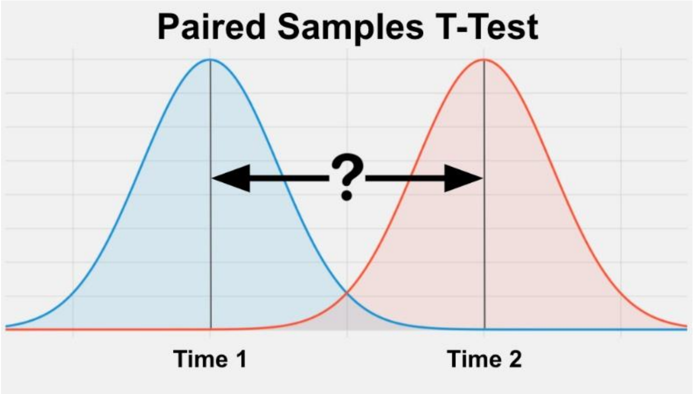
s = sd(x)



$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

s = pooled

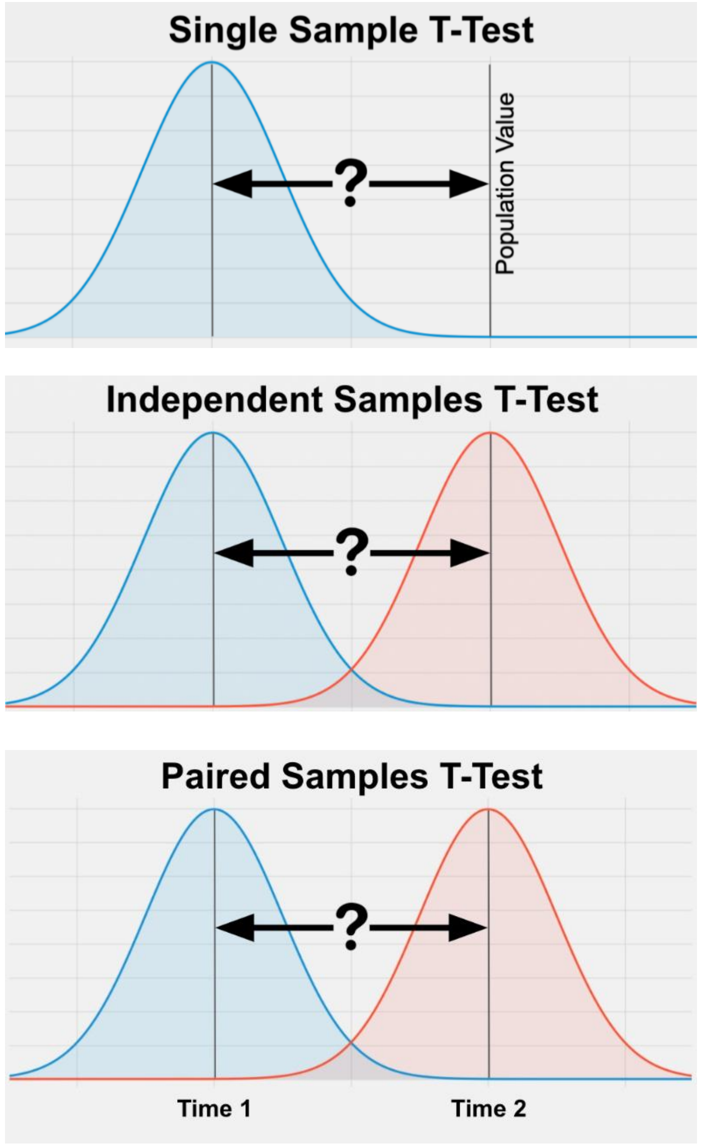
$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$



$$t = \frac{\bar{d}}{s/\sqrt{n}}$$

s = sd(x2-x1)

Diff: T2-T1



Diff: T2-T1

AS REGRESSIONS

$$Y - \text{null} = b_0 + e$$

```
lm( y - mu ~ 1 )
```

$$Y = b_0 + b_1 * D\_treat + e$$

```
lm( y ~ treat_d )
```

$$Y_{it} = a_i + b * D\_post + e_{it}$$

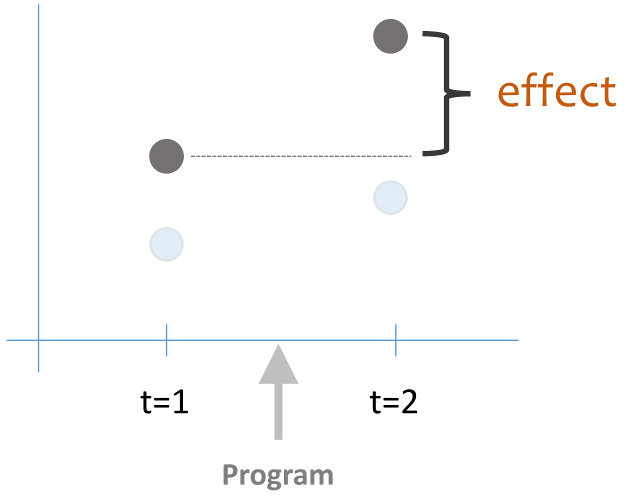
```
lm( y ~ post_d + factor(id) )
```

Connections to other topics:

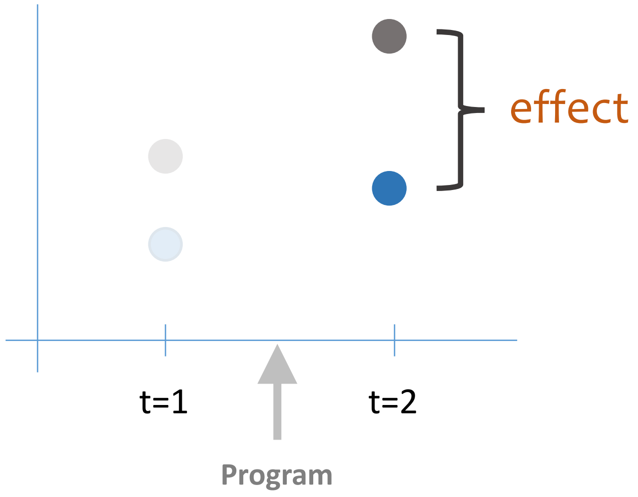
3 valid counterfactuals:

Each variety of counterfactual has a different formula for the program effect estimate.

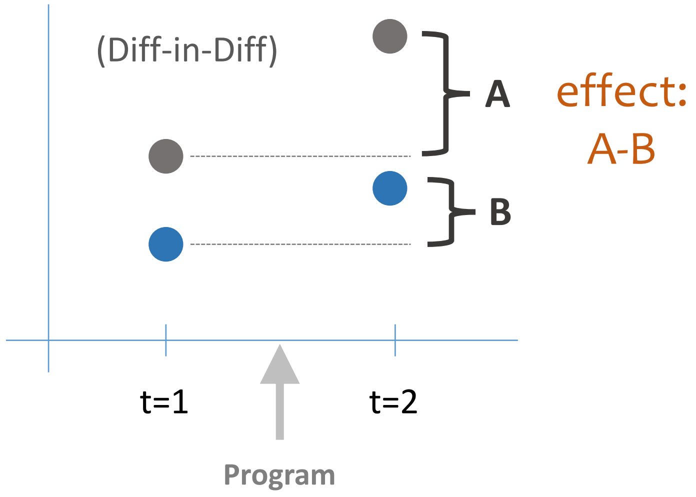
Pre-Post Reflexive
Estimator
effect = T2-T1



Post-Only
Estimator
effect = T2-C2



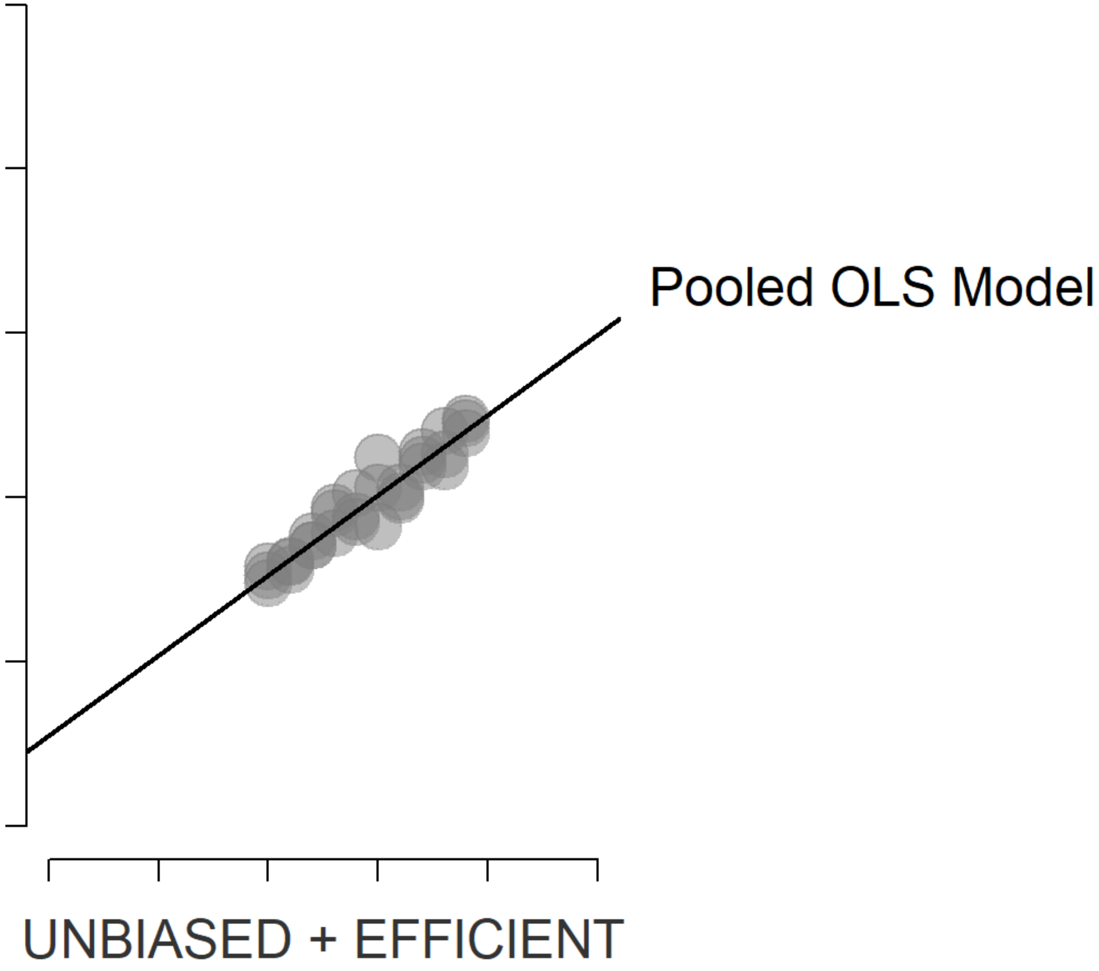
Pre-Post w Comparison
Estimator
effect = (T2-T1) – (C2-C1)



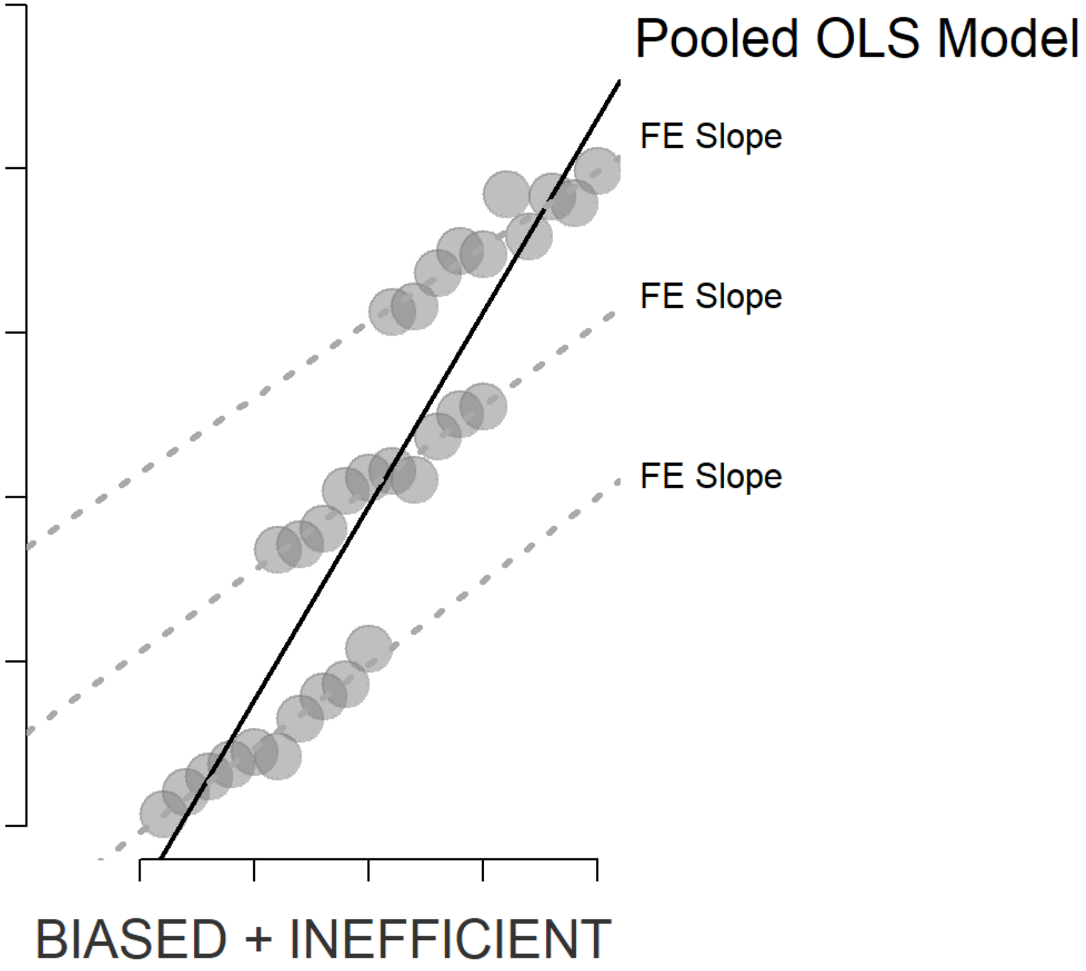
● Treatment Groups, **T1**=before, **T2**=post-program measure

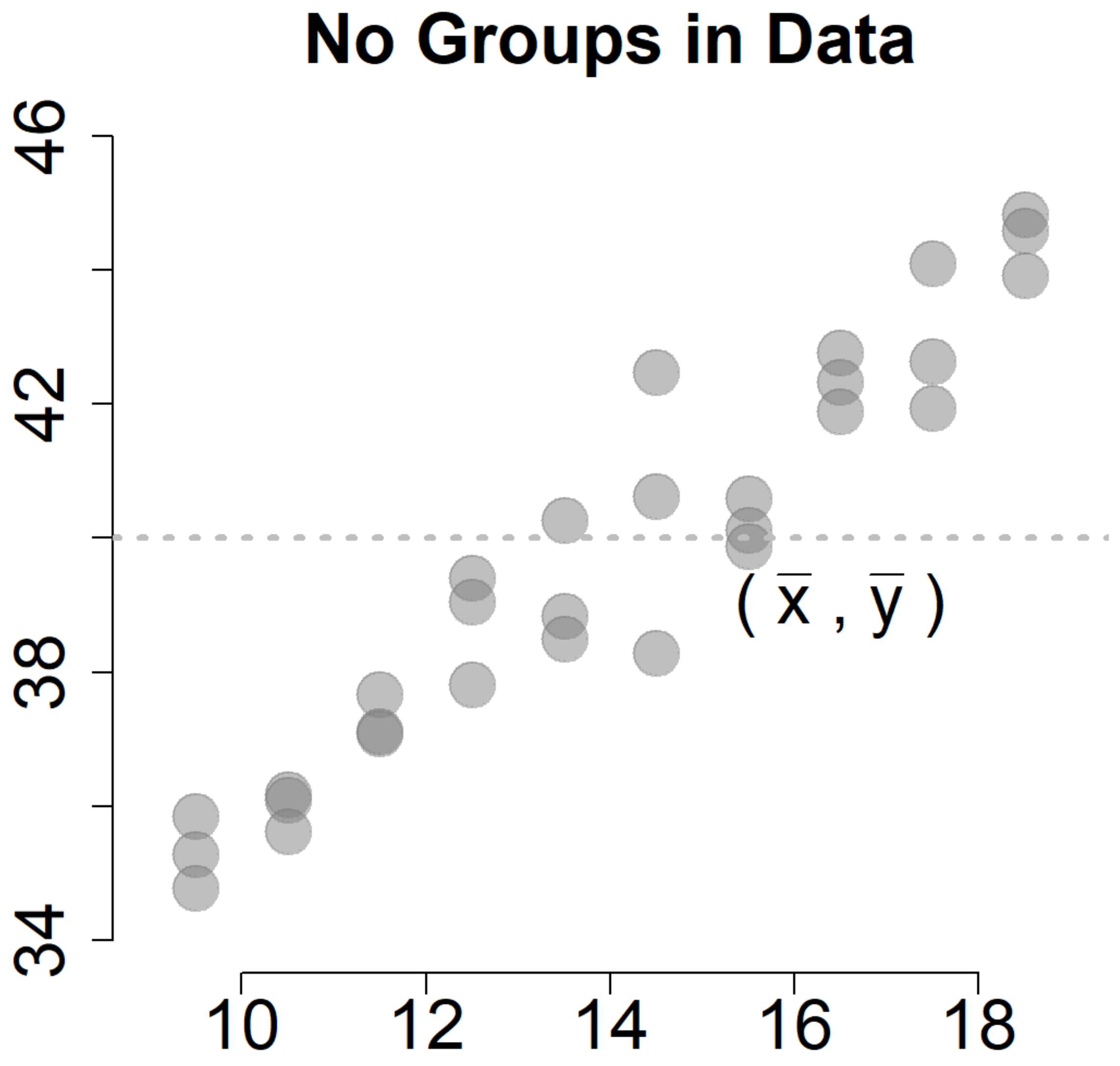
● Control Groups, **C1**=before, **C2**=post-program measure

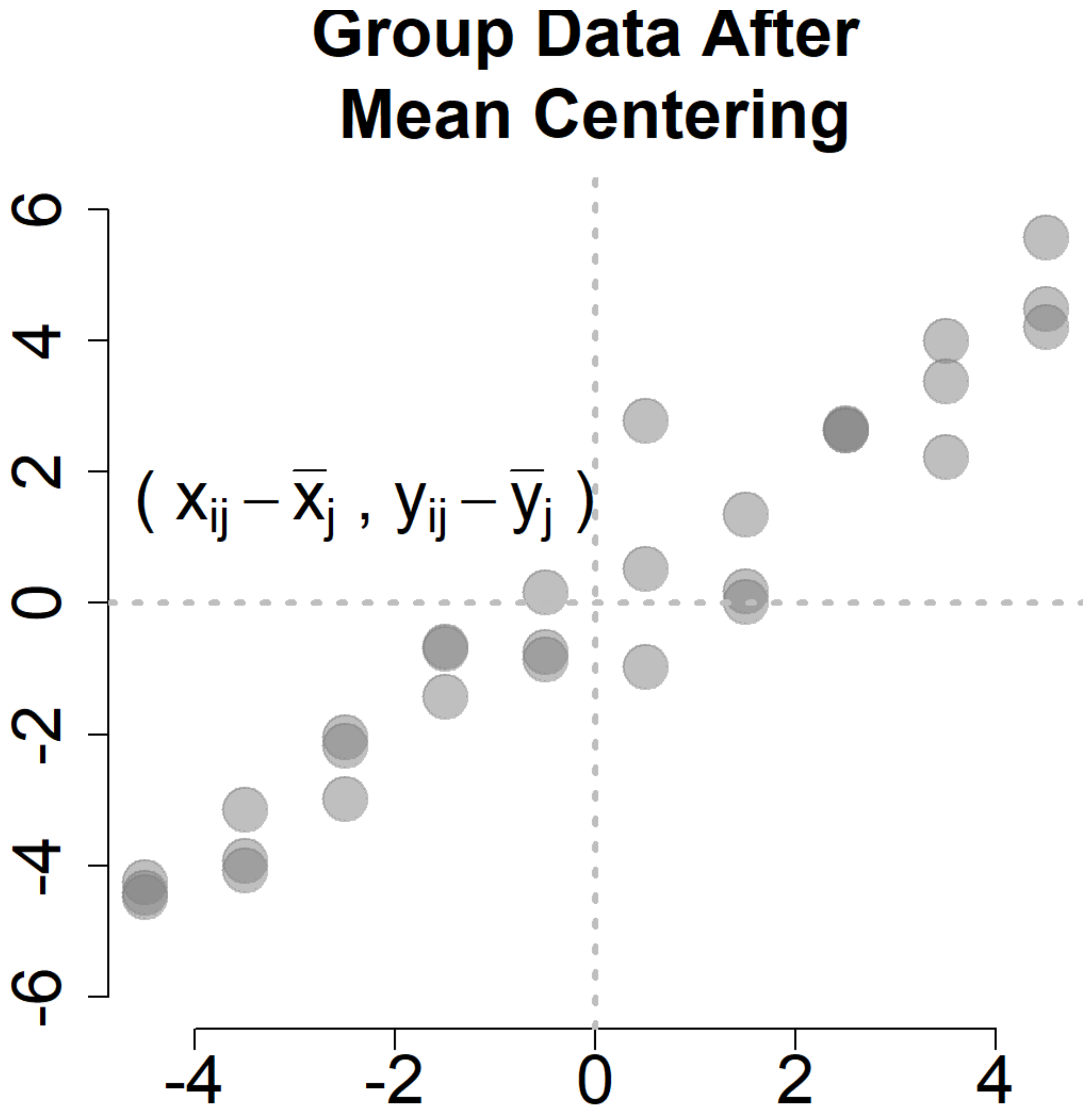
Data with No Groups



Data With Groups







Note, this is just another partitioned regression approach where the control variable is the group ID

META-ANALYSIS

Standardized Mean Difference (SMD) vs. Standardized Mean Change (SMC)

Both the Standardized Mean Difference (SMD) and the Standardized Mean Change (SMC) express results in standardized units, allowing researchers to compare findings across studies. However, they apply to different research designs and interpret different forms of change.

1. Standardized Mean Difference (SMD)

The Standardized Mean Difference measures the difference between two *independent group means* in standardized units. It is used in between-subjects designs, such as comparing a treatment and control group.

Formula: **$SMD = (M_1 - M_2) / SDpooled$**

Where:

M_1 = Mean of treatment group

M_2 = Mean of control group

$SDpooled = sqrt[(SD_1^2 + SD_2^2) / 2]$

Common estimators include Cohen's d, Hedges' g, and Glass's Δ.

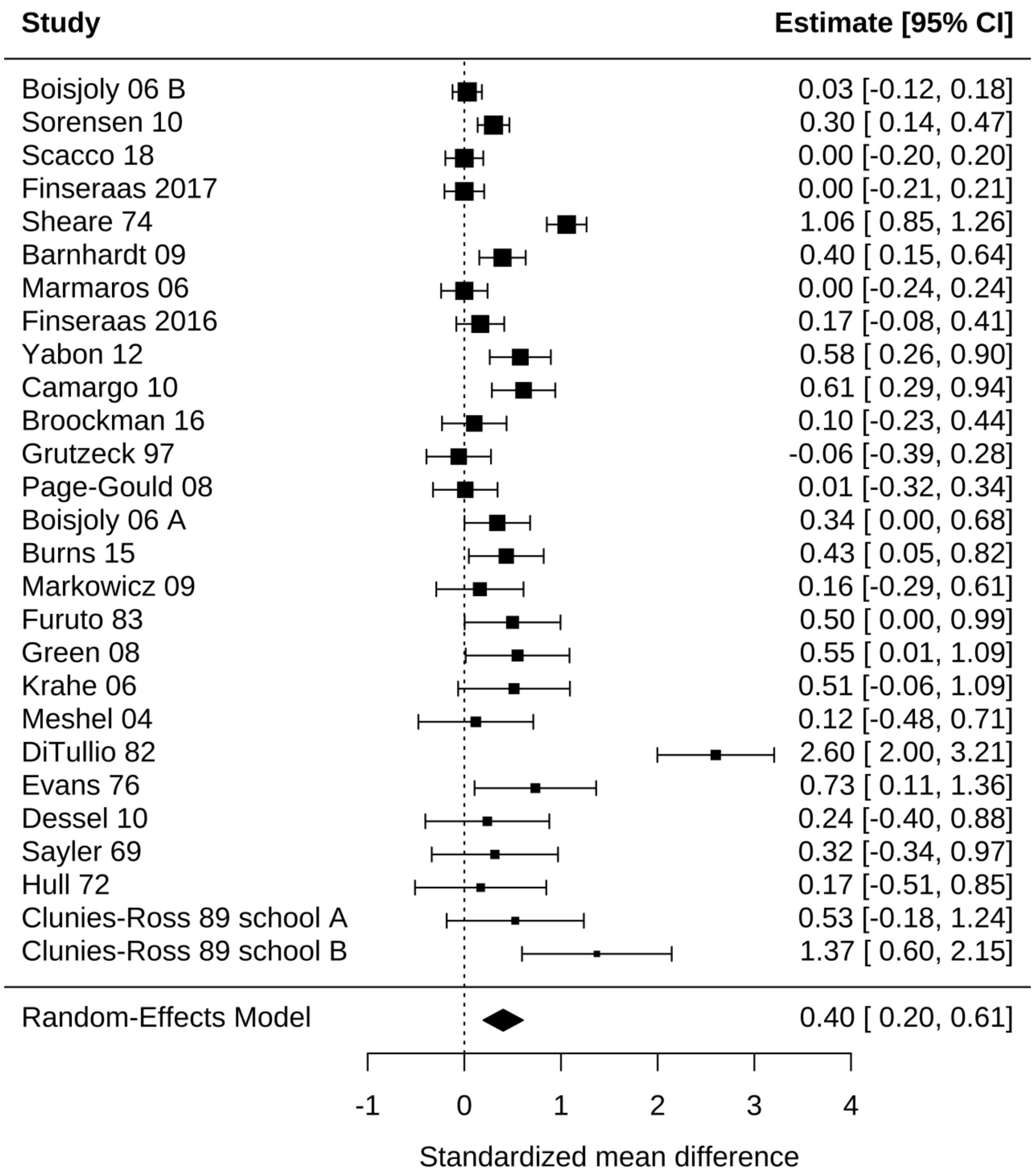
2. Standardized Mean Change (SMC)

The Standardized Mean Change measures the magnitude of change *within the same group* over time, such as pre-test vs. post-test results. It is used in within-subjects or repeated-measures designs.

Formula: **$SMC = (Mpost - Mpre) / Sdpre$**

Common estimators include variants described by Morris & DeShon (2002), such as:

- Change standardized by the SD of change scores (dchange)
- Adjusted versions accounting for pre-post correlations



RESEARCH SYNTHESIS USING META-ANALYSIS

https://experimentology.io/016-meta.html

Figure 1: Various charts and plots common to meta-analysis. https://www.nature.com/articles/nature25753